Age and Economic Growth:
Synthesis and Extensions

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I. Introduction

Fertility declines throughout the developed world have led the governments of many of these countries to express concern and to introduce pronatal policies intended either to raise fertility or to prevent further declines; no government of a developed country in recent years has, in the periodic United Nations surveys, reported a concern that its population growth rate was too high. There are various reasons why these governments are disturbed by the typically sub-replacement fertility levels, but one of the most important is that the resulting age distributions appear to have unfavorable implications, particularly for the burden of supporting the elderly population.

The research project reported here deals primarily with this issue, albeit from a rather abstract perspective. It presents an analytic and critical review of a growing economic literature on the economics of age distributions, and most particularly on the relation of intergenerational transfers to population growth. In addition, many other topics are discussed, including consequences of changing household composition for the allocation of time and goods; consequences of changing population growth, structure and size more generally; the impact of slowing population growth on income distribution; and the existence and magnitude of externalities to child bearing. These topics will be discussed below, and detailed treatments may be found in the attached papers.
II. Age Distribution and Intergenerational Transfers of Goods in a Household Setting

The seminal paper within economics on intergenerational transfers was published by Paul Samuelson in 1958. It established the useful abstraction of an economy in which the only productive factor was labor (so that neither capital nor fixed resources mattered), and the storage of output was impossible, so that total social output had to be consumed each instant. In such an economy the only economic consequence of population growth is from age distributional changes. Samuelson distinguished three age groups. In Lee (1980) this model is generalized to a continuous age distribution. Samuelson's "biological interest rate" result is seen still to hold: a rate of interest equal to the population growth rate would clear the market for intergenerational loaning and borrowing, but would not generally arise from the workings of competitive markets; instead some special institutional arrangements, such as a social security system, are necessary.

A key concept in these models is the age-specific consumption schedule, c(x), and earning schedule, y(x). Although apparently straightforward, we will see below that the c(x) schedule is slippery indeed.

In such models, it is readily shown that the effect of a change in the steady state growth rate, arising from altered fertility, is as follows:

$$\frac{\partial \ln C}{\partial n} = A_c - A_y$$
where \( C \) is the present value of expected life cycle consumption, \( n \) is the population growth rate, and \( A_c \) and \( A_y \) refer to the age at which the average good is consumed (\( A_c \)) or produced (\( A_y \)) in the stable population, respectively (see Lee, 1980). This result builds on Arthur-McNicoll (1978).

Samuelson (1958) viewed the life cycle as starting with adulthood; the prior consumption of adults when they were children was to be viewed as part of their parents' consumption. Therefore children were in effect ignored. In a typical case, imagine people living from birth to age 75, earning a fixed amount from age 20 to age 65 and nothing thereafter, and consuming a fixed amount each year from age 20 to death at age 75. Then in a stationary population, \( A_c = 47.5 \) and \( A_y = 42.5 \), so \( A_c - A_y = 5 \). Thus moving from a growth rate of \( 0 \) to one of \( .01 \) (\( dn = .01 \)) would permit life cycle consumption to increase by five percent \( (.01 \times 5) \). Because production takes place on average at younger ages than consumption, "wealth flows upwards" in Caldwell's (1982) sense, and higher fertility with more rapid population growth is beneficial.

In later work, Samuelson (1975, 1976) took into account savings and capital accumulation; when this is done, the positive transfer effect is counterbalanced by a negative "capital dilution" effect. Samuelson argued that for some optimal population growth rate these would just offset one another. There were technical problems with his analysis (Deardorff, 1976), but a deeper issue was raised by Arthur-McNicoll (1978), who not only achieved a major advance by recasting the Samuelson model in terms of a population with a continuous age distribution, but also pointed out that child consumption costs should be included, and that if they were, the \( A_c \)
would be less than $A_j$, and both the transfer effect and the capital dilution effect would be negative: slower population growth would always be beneficial. Lee (1980) generalized this result in several ways: 1. restrictive assumptions about the utility function are unnecessary; all the action comes from the budget constraint; 2. children's consumption is chosen by their parents, and should be modelled in a household setting; 3. children have a time cost which may reduce the parents' earnings. Each of these points is of some importance. The second one will be discussed in detail in the remainder of this section; the third will be addressed in the next section.

The concept of children's consumption presents many difficulties, but on inspection these are found to be characteristic of the concept of age-specific consumption in general, and not specific to children's. The difficulty is not simply that children do not choose their own consumption (as assumed, for example, by Arthur-McNicoll, 1978, and by Arthur, 1980); it is that children consume together with adults in households. Many aspects of household consumption are subject to increasing returns to scale, and include public or quasi-public goods. Unless people live only in single-person households, it is therefore difficult to say who consumed what. The problem is not merely empirical and practical; it is also conceptual: we don't know quite what it is we want to know. These problems are discussed in Lee (1980), where I believed I had resolved some of them by employing the Tobin (1967) model of intertemporal household consumption allocation, based on the notion of equivalent adult consumers. However, because this approach did not distinguish between average and marginal consumption effects, it did not address the scale economies issue,
and it could be reexpressed in the atomistic form of the original Arthur-McNicoll (1978) result. It did, however, contribute to the proper analysis of the shape of lifetime consumption profiles, by tying childhood consumption to the earnings and decisions of the parents.

In Lee (1984a) I make a thorough attack on the problem by dropping the special assumptions of the Tobin model. In this paper I only assume that some function \( v \) describes the adult consumption received by the household heads, given total household consumption, \( C(x) \) (for head aged \( x \)) and given household composition described by \( H(x,m) \), giving the number of members age \( m \) in a household with head aged \( x \). Thus

\[
v[C(x),H(x,m)] = v(x)
\]

describes adult consumption by age, when \( C(x) \) and \( H(x,m) \) are known. The function \( v \) can be highly nonlinear, and incorporate returns to scale to an arbitrary degree.

Because of these nonlinearities, the analysis must now deal with the allocation of individuals to households, and keep track of the frequency distribution of household composition by age of head. It won't do to allow expected (fractional) numbers of members. Results will depend on the allocation rules chosen, and therefore on the institutional setting.

For the contemporary U.S., it appears roughly appropriate to assume that children live with parents, and adults live with their children until the children reach the age of forming their own households, and that adults live with a surviving spouse, but with no non-spouse adult. Thus the elderly live alone or with a surviving spouse. I do not consider divorce,
or that children may live away from home before marriage. I assume that every couple has \(2+g\) children, and examine the effects of small variations in \(g\), which generate variations in \(n\), the population growth rate, about zero. Note that positive variations in \(g\) correspond to the addition of a fraction of a third child to a household.

The result of the analysis is that:

\[
\frac{\partial \ln C}{\partial n} = A_C - A_Y - 2\alpha A_f,
\]

where \(A_C\) and \(A_Y\) now refer to the age of the head of the household in which the average good is consumed or earned, and therefore they do not depend on the concept of individual consumption at all. \(A_f\) is the average age of childbearing. \(\alpha\) is the present value of the net incremental cost of a third child, as a fraction of \(C\), the present value of life cycle household consumption. (This is similar to equation 19 in Lee, 1980). The important variable \(\alpha\) is defined in terms of consumption costs less earnings, and consumption costs are measured as:

\[
[\partial v/\partial H(x,m)]/[\partial v/\partial C(x)]
\]

where the numerator is the derivative of \(v\) with respect to a third child, conditional on survival status of the parents and two older siblings. Now this ratio, on inspection, is just the increment to total household consumption necessary to restore the adults' consumption, \(v\), to what its level would have been in the absence of the third child.
This concept of child costs is precisely the concept addressed empirically in studies by Espenshade (1973), Olson (1983), and (after some manipulation) Lazear-Michael (1983), following Henderson's methods based on "observable adult goods."

It appears, therefore, that this paper (Lee, 1984a) resolves some troubling conceptual and technical issues in the literature, and in so doing points the way toward more serious empirical assessment of the transfer effects for the first time. This is accomplished through the careful distinction of effects arising from changing composition of households by age of head, and of intergenerational transfers conducted within households at "cut rates," due to economies of scale.

Furthermore, this analysis permits treatment of externalities to childbearing. Suppose couples maximize $U(V[C(x),H(x,m),f])$, where $f$ is their fertility, and $U$ is a function of adult consumption at every age. The constraint is that the present value (at discount rate $n$) of household consumption equal household earnings. It is easily shown that adults will choose $f$ such that $\frac{\partial U}{\partial f} = \lambda C$, or that they will have children until the marginal utility of an additional child just equals the net utility cost of a child, $\lambda C$. As individuals, they do not take into account the fact that $n$ depends on $f$. A social planner, however, would take this into account, and would instead mandate fertility such that:

$$\frac{\partial U}{\partial f} = -\lambda C[(A_C - A_Y)/(2A_C) - \alpha].$$

This will be greater or less than the individually optimal fertility depending on whether $A_C$ is greater or less than $A_Y$. 
Transfers down, from parents to children, take place at least in part within the family (and as modeled here, entirely so). Transfers up, however, do not come specifically from one's own children; they come from a high interest rate that rewards heavily the saving "for retirement"; if fertility were lower, then either the rate of return to K would fall or the money old people held would lose value through inflation. Thus the costs of a higher fertility, in terms of supporting children, are internal to the family; the social benefits (in this restricted context) are externalities. So if $A_C > A_Y$, one would expect a sub-optimal rate of population growth to prevail (ignoring other expectations -- fixed and reproducible resources, which would actually tip the balance in the other direction).

In the paper, I present data-based estimates of $A_C$, $A_Y$ and $\alpha$. $A_C$ and $A_Y$ are based on the 1972/73 CES, while $\alpha$ is based on the Lazear-Michael (1983) study, together with Mason (1975). Public costs for health and education are added to these estimates of private costs. It appears that $A_C - A_Y$ is about 2.6, while $\alpha$ is about .035. If the average age of fertility is 28.6, then $2A_\alpha = 2.0$, and $A_C - A_Y - 2A_\alpha = .6$. From this we would conclude:

1) If the population growth rate were increased from .00 to .01, adults would increase their life cycle consumption by .6 percent.

2) Taking into account the direct marginal utility from children, which in the neighborhood of laissez-faire fertility levels would just offset the marginal child costs born by the household, such an increase in population growth rates would effectively raise the life cycle consumption of adults by 2.6 percent.
The effect described in 1) is negligible; that in 2) is not. Therefore, this analysis comes down somewhere between Samuelson and Arthur-McNicoll. The formal result for $\ln C_n$ in the neighborhood of laissez-faire equilibrium resembles Samuelson's (1958) results, but in fact is different since the $A_C$ and $A_Y$ refer to households rather than individuals, and thus include the effects of children on life cycle consumption patterns to whatever extent is empirically appropriate.

III. Broadening the Scope: Intergenerational Transfers of Goods and Time

But the production of physical goods occupies a relatively small portion of the total life cycle quantity of time at an individual's disposal. For example, if everyone lived to the age of 70, and worked at market tasks from the age of 20 to 65, with men working 40 hours per week and women 20, then time devoted to market work would account for only about 11.5 percent of the total time budget. Even if we deduct ten hours a day as a basic minimum necessary for biological maintenance (eating, sleeping, etc.), and calculate the fraction of market time out of the remainder, it is still only about 20 percent of this "discretionary" time. Might, then, this exclusive focus on the intergenerational flows of market goods conceal some other important aspects of intergenerational resource flows more broadly construed? Here I will attempt to develop a fuller accounting of time use, and analyze its implications in an intergenerational context.

In Lee (1983) I develop in some detail the view of a household economy in which allocations of goods and time are strongly affected by the age-sex
composition of the household, and show that a simple theoretical framework can link and synthesize much literature dealing with such effects. For present purposes, all we need is the notion of a household which allocates the time of each of its members among leisure, home services, market labor, and education. Members derive utility from their own leisure, and from the consumption of home services and market goods. Individuals can enjoy home services and market goods produced by others, but only their own leisure; thus only home services and market goods may be transferred intergenerationally. Households, and society, operate subject to a budget constraint requiring that all consumption be produced, and limiting the total time available to 24 hours per day. Individuals of different ages have differential efficiency per unit time, and this efficiency schedule is maintained by inputs of time and market goods to education.

All the discussion of the previous part, indicating the importance of distinguishing between the "within household" transfers and the "between household" transfers applies in this part as well. I discussed above the way in which the net costs of a third child could be calculated for market goods, drawing on the excellent study by Lazear and Michael. Unfortunately, there is no comparable study for the time costs of children. While some studies, such as Turchi (1975), estimate time costs of children by birth order, and reveal incremental changes to be about twice for the first child what they are for subsequent children, these studies do not make it possible to calculate the compensating increase in time (or income) which would leave the adults as well off (in terms of leisure, say) as they were before the incremental child.
For this reason I have only done the "between households" part of the numerical calculation. This will reveal whether or not there are significant age-distributional externalities to fertility, but will not reveal the full social and private costs of fertility and population growth.

The basic result is that:

$$\frac{\partial \ln C}{\partial n} = A_E - \{A_{wL} + A_{wst} + A_{wh} + A_{wc} + A_{wsm}\}$$

Here the w's are weights equal to the share of the present value of the household's life time wealth spent on each item, and therefore the w's sum to unity. The subscripts refer to the following age schedules:

- e(x) efficiency of work hours
- t(x) consumption of leisure
- st(x) time spent receiving schooling
- h(x) consumption of home services, measured in efficiency units
- sm(x) market goods devoted to schooling

The $A_E$ term represents the average age of potential production, if everybody worked 24 hours per day, weighted by efficiency. The quantity in brackets is just the average age of generalized consumption of time, which is the weighted average of the average of each type of time use. These schedules all refer to total quantities in the household with head age x, so they reflect the presence of two each of adults and children. The C on the left is the present value of the generalized use of time, not all of which has direct consumption value.
Numerical results were obtained using ISK's (of the University of Michigan) time use survey, together with cost-of-child estimates based on Olson (1983), adult consumption and earnings from the CES (1972/73), and various sources for public expenditures.

The numerical results are summarized in Table 1, which is extracted from Lee (1984b). The age profiles are shown at the bottom of the table, all converted into dollars of daily household consumption, using age specific efficiency weights and an appropriate real wage.

The top panel shows the average ages (A_C = 47.318), and so on), the shares (w_C = .10388), and the product (A_C x w_C = 4.91556). From this calculation, the average age of generalized consumption (46.684 = sum of products) exceeds that of generalized "full production" (46.337) by .35 years. This suggests that an increase in the population growth rate from .00 to .01 would increase generalized life cycle consumption by .35 percent, a tiny amount.

But note that this is as a proportion of total household life cycle wealth, which is dominated by the non-transferable commodity leisure, which alone accounts for 75 percent of household consumption. Reexpressed in terms of the transferable consumption commodities, that is, market goods and home time services, the proportionate increase is about 4 x 35 = 1.4 percent, a somewhat larger amount. Also note that this does not include the incremental time and goods cost of children born by the household, since these are assumed to be offset by an equal marginal utility of children in laissez-faire equilibrium. This calculation does not yield results very different than those of the previous part; external transfer effects are still positive, but are now very small.

All these calculations looked only at the age-distributional effects of fertility and growth rate changes. It is beyond the scope of these
models to consider the role of fixed natural resources (although see Samuelson, 1976, and a sketchy discussion in Lee, 1980). The role of capital accumulation, however, is easily incorporated (see Lee, 1980). If we consider the consequences of capital dilution across golden rule (optimal saving) steady states, then the term $-k/c$ is added to $\delta \ln C/\delta n$, where $k/c$ is the ratio of capital to consumption. In Lee (1980) I suggest this is probably in the neighborhood of $-7$ to $-20$, and probably swamps the intergenerational transfer effects (which I have reported above as in the range zero to $+2.4$ in comparable units.) But the true role of capital in economic growth is problematic, while the intergenerational transfers and age distribution effects seem quite concrete.

Perhaps the more general point to make is this. By all reckonings, the transfer effects appear quite small. A "once-for-all" increase in life cycle consumption of $.4$ to $2.6$ per cent consequent on an increased population growth rate of one percent per year is scarcely larger than the fruits of a single year's increase in productivity -- before the stagnant 1970s, at any rate. Yet an increase of one percent in the population growth rate would have broad social and economic repercussions, compared to which the intergenerational transfer effect would surely look very small indeed.

IV. Population Growth and income Distribution

In a related project, that is analytically similar to the previous in several respects, David Lam (forthcoming) investigated the effects of changing population growth on the distribution of income. (This paper won the Dorothy Swaine Thomas Award in 1983). For each age group there is a mean level of income, and there is also some degree of variance about that mean. When fertility changes, the growth rate and age distribution also change, and the different population age groups contribute with changed
weights to overall inequality. Most scholars have expected that the net effect of more rapid growth and a younger population would be to increase inequality, but some have argued that growth would reduce it. Lam derives an exact mathematical decomposition of the effects, and evaluates the effect of changed fertility (across steady states) on inequality using U.S. and Brazilian data. For the U.S., there is a fairly strong effect in the direction most scholars have expected: more rapid growth leads to greater inequality; moving from \( n=0.0 \) to \( .01 \) would increase inequality (the variance of the logarithm of income) by six percent. With Brazilian data, however, the opposite occurs, and inequality is very slightly diminished by more rapid growth. Further investigation reveals that this is also true with U.S. data if the 15-19 year olds are excluded.

The kinds of effects just described would lead to differences in measured income, but no effects on the inequality of life cycle income profiles. In this sense, the analysis suggests that when measuring income distribution, age compositional effects should be controlled, for in some circumstances they may matter.

This paper also addresses the more general issue of how to analyze the changing distribution of population characteristics when growth rates change, extending the previous analysis of changing prevalences or levels, to higher moments.
### Table 1

**Population Growth Rate:** \(0.000\)

\[
\text{Wage} = 0.026
\]

**Total Wealth = 4839157.**

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Appendix A. Bibliography of Papers Written on This Project


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